

A Dynamic Implementation of the Markowitz Portfolio Allocation Procedure

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ABSTRACT

We evaluate two dynamic (quasi-continuous time) implementations of versions of the Markowitz procedure, using “iShares” and “Spiders” to capture an international opportunity set. The standard Markowitz procedure is supplemented with an alternative which *a priori* has the potential to outperform the usual procedure in “bear” markets. The alternative situations are distinguished with the “tangent point criterion”, which we suggest as a standardised measure of the overall risk premium. The Markowitz portfolio allocations are often found to be unrealistically extreme, so we also consider “Lintner” rescaled portfolios as well as “naive” and “world” portfolios. A particular “zero-investment” procedure is introduced for the dynamic evaluation of portfolio performance.

For this data the standard Markowitz procedure, using inputs estimated *ex post*, produces superior risk adjusted *ex ante* returns, suggesting that there are economic benefits contained in *ex post* data on mean returns and their covariances, and that, even in a quasi-continuous time implementation, the Markowitz procedure forms a baseline standard against which other methods can be measured.

1. Introduction

There has recently been a resurgence of interest in Markowitz's (1952) foundational portfolio optimisation methods over alternatives such as index models. The latter require fewer parameter estimates, which can consequently be made with greater precision from limited data, but portfolios constructed in such a way must be sub-optimal (with respect to mean-variance optimisation) by comparison with Markowitz allocation, and with the availability these days of large, reliable, data sets the issue of precision of parameter estimation is no longer of such concern. Jagannathan and Ma (2003) report on an extensive investigation which compares covariance estimates constructed from daily or monthly returns data, using Markowitz or index models, for 500 stocks on the NYSE and AMEX from April 1968 to April 1988. The use of monthly data resulted in singular covariance matrices which they overcame by using shrinkage estimates. With daily data they encountered no such problem and the corresponding portfolio performances were hardly affected by comparison with shrinkage methods. For this data, the one-factor model performed rather poorly.¹ Jagannathan and Ma (2003) conclude by suggesting that further investigation into the use of daily data in Markowitz modelling may prove profitable.

¹ In an earlier investigation, Chan, Karceski and Lakonishok (1999) found that the Fama and French (1992, 1993) 3-factor model improved out-of-sample forecasts of the minimum variance portfolio in a particular data set. But we are concerned with the Sharpe ratio-optimal portfolios.

In this paper we take up that suggestion and report on a practical evaluation of Markowitz portfolio allocation in a setting which approximates a continuous time implementation. The focus will be on attempting to develop and evaluate realistic dynamic procedures which can be put into practice using only concepts, resources and techniques currently well understood by practitioners. The “continuous time” aspect (actually, using daily data) is an attempt to utilise available data to the maximum possible extent; again, consistent with practical considerations, we take into account transaction costs associated with the quasi-continuous implementation. Keeping these objectives in view helps reveal some of the features, good and bad, of the Markowitz methodology as it can be implemented with modern technology.

In addition, we wish to draw attention to an aspect not usually (formally) considered, namely, the existence or not of a tangent point to the efficient frontier generated by the set of risky assets. The existence of the tangent point depends on the sign of the *standardised risk premium*, as we define it later, in terms of a quantity we call the *tangent point criterion* (or *TPC* for short), obtaining in the market. Consequently this issue is of special relevance in bear market conditions. It is perhaps not generally appreciated by practitioners that the usual Markowitz procedure attempts to cope with the case when a tangent point does not exist by shorting the entire “optimal” portfolio (which in its original form is the opposite of optimal in this situation) and investing the proceeds in the risk-free asset (assumed to be available). A decision on the proportionate allocation of funds to risky or risk-free assets has then to be made, and, furthermore, at the time when the standardised risk premium

changes sign from positive (the Markowitz optimal case) to negative, a massive reallocation of resources would seem to be required, which, we might hypothesise, should add greatly to transaction costs. These considerations suggest that in a practical implementation the optimality properties of the Markowitz procedure might be outweighed by the costs associated with implementing it, and lead us to consider a second method based on an analysis by Maller and Turkington (2002), who showed how to form the portfolio with maximum Sharpe ratio *when the risk premium is negative*. This procedure does not require the large rebalancing apparently needed by the standard Markowitz method, and thus *a priori* it would appear to be more frugal in terms of transaction costs. On the other hand, ignoring transaction costs, it is suboptimal with respect to the standard Markowitz procedure.

To distinguish these two variants of the Markowitz procedure, we will refer to the standard Markowitz procedure, together with shorting of the whole portfolio and investment in the risk-free asset when the standardised risk premium is negative, as the *augmented (standard) method*. The second method will follow the standard Markowitz procedure when the standardised risk premium is positive ($TPC > 0$), and the Maller and Turkington method when it is negative ($TPC \leq 0$). We will refer to this as the *combined method*.

There are essentially two ways to compare allocation procedures and strategies on data, both having their pros and cons. One way is to *simulate* data, and apply and evaluate procedures in a situation where we know what the answer should be, at least, within calculable limits. There are great advantages to knowing “what the answer should

be”, of course, but the drawback to this method is that simulated data cannot allow for the vagaries and variations of real-world situations. Thus it can never be more than a kind of controlled exploration which helps to determine limits of variation and to highlight problems with procedures that occur in a “best-case” situation where by decree we have precise knowledge of the parameters of a randomly varying process. As an alternative, we can take a *clinical study* approach, whereby we apply our procedures to a particular set of data over a specific period of time. In this way we capture a real-life situation, but the downside is that we must trade off generality – the inferences we wish to make to wider populations are always problematic and open to an unavoidable criticism of the specificity of the particular situation we analysed. We can alleviate this to some extent by selecting as representative as possible a sample of assets, with the intention that inference from these is less subject to extrapolation errors than that from a more specific set might be. This is the path we chose to adopt in the present research.

In this paper, the performances of the two types of Markowitz portfolios, and some variants of them based on “smoothing” ideas to be introduced later, along with two other benchmark portfolios (the “naive” and a “world” portfolio) are compared on an *ex ante* (out-of-sample) basis, using daily data on a series of “iShares” and Standard & Poor’s Depository Receipts (SPDRS, or “Spiders”). In all, we consider 6 versions of portfolio allocation, along with the World Index as the 7th portfolio. The dynamic (continuous time) aspect is approximated with a daily rolling window allocation procedure for which

analysis is easily carried out and can be justified by martingale methods.² The data we analyse is “global” and not specific to a particular country or industry or other subgroup within a country. It is, perforce, restricted to a certain period of time, but this period encompasses both clearly defined bull and bear markets and situations in between, and so provides a good range of outcomes on which to evaluate our procedures.

Our results cast considerable doubt on the “primacy” of the passive management style often advocated by efficient market proponents. They show that the *ex ante* performances achieved by our optimization strategies dominate passive alternatives. However, we find that the portfolios generated by the augmented and combined Markowitz methods are quite unrealistic in that they require extreme long or short positions to be taken in some stocks at some times. This leads us to suggest some *ad hoc* smoothing techniques which are intended to damp down the extreme positions. While they do indeed accomplish this to a

² It should be noted that our analysis is quite distinct from what is currently understood in the financial literature as “continuous time portfolio analysis”. The large literature in this area is concerned with the setting up and investigation of (usually) *self-financing* portfolios, almost always with the objective of implementing/evaluating option pricing methodologies. While the procedures for defining these kinds of portfolios are often also based on a variant of the Markowitz paradigm, we, by contrast, are concerned with the investment opportunities *per se* afforded by Markowitz methods, and our allowable portfolios are not restricted to those that are self-financing.

greater or lesser degree, they also eliminate to a large extent the gains found by the original methods. It appears that the baby goes out with the bathwater. Nevertheless, overall, the analysis suggests that historic data contains economically valuable information in the quasi-continuous-time scenario we examine. The implication is that, with technology that is now affordable and efficient, investors should consider optimizing to beat the benchmark – not just to track it.

Before proceeding further, the reader might like to refer to Appendix 1, where we give a brief review of the Markowitz procedure, with emphasis on the existence or not of a tangent point, define the *TPC* statistic, and detail the “augmented” and “combined” Markowitz methods. These are essential aspects of our approach. The structure of the remainder of this paper is as follows. Section 2 introduces the data we will use, and the dynamic empirical methodology and portfolio definitions are set out in Section 3 (and in a little more detail in Appendix 2). Methods of evaluation, including the “zero-investment” procedure, are in Section 4, while results concerning *ex ante* predicted returns for Spiders and iShares are in Section 5. (Some statistical considerations are in Appendix 3.) Their implications are discussed in Section 6, and Section 7 concludes the paper.

2. Data

To conduct a realistic study on diversification requires a tractable number of assets that can be analysed without having to consider possible complicating factors introduced by institutional peculiarities. Domestic data sets usually include a very large number of

assets (the number of tradable securities on a domestic market) and a smaller number of portfolios based on those assets, such as industry portfolios, which may not be tradable. The number of international assets readily tradeable is smaller, but national indices are not, in themselves, readily fungible unless traders use derivatives based on them. However, futures and options are highly levered, and, in the case of futures, subject to margin requirements, so that returns experienced by investors in such instruments may be exaggerated compared to those experienced by investors in the underlying.

The dominance of an efficient frontier derived using international assets over frontiers where the opportunity set is confined to a particular domestic market is of course to be expected, theoretically. The practical benefits of such international diversification are, however, arguable. Grubel (1968), Levy and Sarnat (1970), Solnik (1974) and Lessard (1976) provide early and persuasive support in favour of international, rather than purely domestic, diversification. More recently, Black and Litterman (1991), Hatch and Resnick (1993) and Michaud, Bergstrom, Frashure and Wolahan (1996) find that international investment improves, or may improve, the return/risk trade off vis-à-vis portfolios restricted to U.S. assets. On the other hand, Ho, Milevsky and Robinson (1999) and Hanna, McCormack and Perdue (1999) argue that US based investors may not benefit from international diversification, while Shawkey, Kuenzel and Mikhail (1997) find mixed evidence for any such benefits, arguing that benefits, or the lack thereof, found in many studies may be largely driven by the particular (idiosyncratic) methods employed.

Our analysis will throw some light on these discussions by focussing on a highly

representative set of international assets. We decided to analyse iShares (discussed in Olienik, Schwebach and Zumwalt, 2001) and Spiders (discussed in Elton, Gruber, Comer and Li, 2002), which are instruments traded on the American Stock Exchange that closely mimic Morgan Stanley International country indices and the S&P 500 index respectively, and provide both tractable and realistic assets with which we may conduct a clinical study.

Our data record commences on March 19, 1996 (the first date on which all 17 iShares were trading) and concludes on March 28, 2002, giving a total of $T = 1517$ trading days. We have $N = 18$ assets (17 iShares plus the Spiders, representing the US). As risk-free asset we use the US Federal Funds (effective) middle rate.

Rolling 30-day windows are used for portfolio specification and evaluation; there are a total of $T - 29$ of them possible in our data.³ A feature of our approach is an emphasis on whether the standardised risk premium, as we define it, is positive or negative. The conditions where the standard Markowitz procedure may be sub-optimal appear with surprising frequency, thus justifying our focus on this aspect (see Section 3 and Figure 1a).

Next, we turn to the methodology used for the empirical investigations, and the results.

³ For comparison purposes we also used 60 and 90-day windows. These analyses yielded results qualitatively similar to the 30-day windows, so we will not report them.

3. Dynamic Implementation Methodology

Let \mathbf{S}_t denote the N -vector of the $N = 18$ asset prices as observed from market closing prices on day t , $t = 0, 1, 2, \dots, T = 1517$. The daily returns are calculated from close to close. Thus $\tilde{\mathbf{R}}_t = (\mathbf{S}_t - \mathbf{S}_{t-1})/\mathbf{S}_{t-1}$ (with the divisions taken component-wise) is the *discrete raw return vector* on day t , $t = 1, 2, \dots$. From it, subtract the prevailing risk-free rate, r_t , to get the *excess return vector* \mathbf{R}_t . Let μ_t and Σ_t be the expected value and covariance matrix of \mathbf{R}_t , considered as a random variable; in general, we allow for them to change with time. On a given day t (our “operating day”), we estimate μ_t and Σ_t by the sample mean and covariance matrix based on the previous 30 trading days’ observations:

$$\hat{\mu}_t = \frac{1}{30} \sum_{s=t-30}^{t-1} \mathbf{R}_s,$$

and similarly for $\hat{\Sigma}_t$. We begin this series of estimates on day 31 and conclude it on day 1517; thus, the first window extends from March 19, 1996 – April 30, 1996, from which we calculate $\hat{\mu}_{31}$ and $\hat{\Sigma}_{31}$, then the window is rolled forward one day to get $\hat{\mu}_{32}$ and $\hat{\Sigma}_{32}$, etc., and we continue this process till we finally obtain $\hat{\mu}_{1518}$ and $\hat{\Sigma}_{1518}$. Because of the smoothing involved in the averaged back portfolios (see later in this section), there is a further lag of 5 days involved, and so our “operating period” extends from May 7, 1996 – March 28, 2002.

The observed *TPC* for day t , denoted \widehat{TPC}_t , $t = 31, 32, \dots, 1518$, is calculated from $\hat{\mu}_t$ and $\hat{\Sigma}_t$ as specified in Eq. (1) of Appendix 1. It is plotted in Figure 1a, and we immediately see that $TPC < 0$ very often occurs, in fact, about 40% of the time in our data.

[FIGURE 1 ABOUT HERE]

On each of days $t = 31, 32, \dots, 1518$ we also perform the two Markowitz optimisations (standard and combined methods) as outlined in Appendix 1, and find the allocations for the augmented and combined portfolios. Call these $\mathbf{x}_t(A)$ and $\mathbf{x}_t(C)$, respectively.

As a measure of the fluctuation of a portfolio allocation \mathbf{x}_t with components $x_t^{(i)}$ at time t (where \mathbf{x}_t is one of the two versions listed above, i.e., $\mathbf{x}_t(A)$ or $\mathbf{x}_t(C)$), we calculate

$$L_t := \sum_{i=1}^N |x_t^{(i)}|.$$

The notation L is supposed to be suggestive of *Lintner Scaling* (Lintner 1965, p. 21), and we will refer to L_t as the *Lintner Score* of a portfolio allocation.

Preliminary examination of the realised values of these measures for our data reveals some interesting features. Figures 1b and 1c show the values of L_t over the entire operating period for the augmented and combined methods, respectively. There is striking variation over time, and the portfolio fluctuations as measured by L_t are enormous.

We also standardise $\widehat{\Sigma}_t$ to obtain the sample correlation matrix, \widehat{C}_t , on day t , and calculate the determinants, $D_t = \det(\widehat{\Sigma}_t)$ and $\det(\widehat{C}_t)$. Figure 1d shows the variation in the determinant D_t with time; it is not closely related either to fluctuations in TPC_t or in L_t , as is seen by comparing Figures 1a, 1b, 1c and 1d.

The portfolio allocations measured in Figures 1b and 1c are so extreme as to be quite unrealistic in practice. Recall that we allow short sales, so the allocations are not bounded in magnitude. We will implement and evaluate these “raw” allocations, but the magnitude

of L_t observed leads us to suggest some modifications which are intended to smooth out the extreme components of the risky portfolios and to attempt to come up with a realistic strategy that avoids excessive transaction costs. The portfolios, \mathbf{x}_t , once modified, will be denoted by \mathbf{w}_t . Those to be investigated here are:

- *Averaged Back Portfolios*. A running average over the last ℓ days ($\ell = 5$ turns out to be reasonable) is taken of the \mathbf{x}_t :

$$\mathbf{w}_t = \frac{1}{\ell} \sum_{s=t-\ell+1}^t \mathbf{x}_s;$$

- *Exponentially Weighted Back Portfolios*. Place most weight (95%, say) on the historical average of the \mathbf{x}_t , and only weight today's new allocation by 5%;⁴

$$\mathbf{w}_t = 0.05\mathbf{x}_t + 0.95\mathbf{w}_{t-1};$$

- *Lintner Scaled Portfolios*. For any given portfolio, define a rescaled one by

$$\mathbf{w}_t = \mathbf{x}_t / L_t.$$

We will apply this procedure both to the augmented and combined methods.

Of the above modifications, the most drastic smoothing is the Lintner rescaling, which ensures that the modified allocations lie between -1 and 1 (and the Lintner Score of a Lintner rescaled portfolio by our definition is equal to 1).

In summary, we investigate the following six versions of portfolio allocation strategies (see also Appendix 2):

⁴ Variants of these weights were tried, but 5%/95% gave the best results.

- (a) *Augmented Portfolio*
- (b) *Combined Portfolio*
- (c) *Averaged Back (Augmented) Portfolio*
- (d) *Exponentially Weighted Back (Augmented)Portfolio*
- (e) *Lintner Scaled (Augmented) Portfolio*
- (f) *Naive Portfolio* (The equal-weighted index of iShares and Spiders).

To these, we also add the:

- (g) *World Portfolio*, as represented by the DataStream World Price Index.

4. Zero-Investment Evaluation Procedure

Let $\tilde{\mathbf{R}}_{(a,b)} = (\mathbf{S}_b - \mathbf{S}_a)/\mathbf{S}_a$ denote the vector of (observed) raw returns of the risky assets over the period (a, b) . The 30-day *ex post* return on a portfolio specified by the allocation \mathbf{w}_t on operating day t is then given by

$$\tilde{R}_t^{(ep)} = \mathbf{w}_t' \tilde{\mathbf{R}}_{(t-30, t-1)}. \quad (6)$$

Effectively, an allocation decided on at day t (and consequently, based on the returns on days $t - 30, \dots, t - 1$) is put in place at time $t - 30$, and accrued returns are calculated at day t . An *ex post* evaluation of portfolio performance is thus backward-looking – it evaluates an allocation on the same data which was used to find it. For this reason *ex post* comparisons are of little interest from a practical point of view. The standard method is always optimal (achieves the maximum Sharpe ratio) when applied to the data from which it was calculated, and consequently can be expected to dominate other methods *ex post*.

This certainly turns out to be the case in our data, and we will not discuss *ex post* results further in this paper.

Ex ante or out-of-sample comparisons are much more interesting. *Ex ante* returns are forward-looking, telling us how an existing portfolio performs in terms of future returns. A special kind of *zero-investment procedure* will be used for the *ex ante* evaluation. Given an allocation, \mathbf{w}_t , decided on for operating day t , as discussed in the previous section, let $W_t = \mathbf{i}'\mathbf{w}_t$ be the sum of the components of \mathbf{w}_t (\mathbf{i} is an N -vector of 1s). After modification as per the preceding section, W_t need no longer equal 1; in fact, $W_t = 1$ for the combined and naive portfolios, but for the augmented Markowitz portfolio, $W_t = \text{sgn}(TPC_t)$, and for the Lintner rescaled portfolios, $|W_t| \leq 1$, and, usually, $|W_t| < 1$.

To implement the zero-investment procedure, the portfolio specified by \mathbf{w}_t is purchased using day t 's closing market prices (day $t + 1$'s opening prices), as follows:

- An amount $\$W_t$ is invested in the risky portfolio according to the allocations \mathbf{w}_t ;
- An amount $-\$W_t$ is invested in the risk-free asset.

The return on this investment (positive or negative) is calculated using day $t + 1$'s closing prices. The return from the risky portfolio is thus $\mathbf{w}_t' \tilde{\mathbf{R}}_{(t,t+1)}$, while the risk-free return is $-r_{t+1}W_t$. The return for day $t + 1$ from the *ex ante* procedure is the sum of these two:

$$R_{t+1}^{(ea)} = \mathbf{w}_t' \tilde{\mathbf{R}}_{(t,t+1)} - r_{t+1}W_t = \mathbf{w}_t' \left(\tilde{\mathbf{R}}_{(t,t+1)} - r_{t+1}\mathbf{i} \right). \quad (7)$$

It is thus the excess return on the investment (and also the value of the investment at the close of day $t + 1$, since we start from a zero position). We call this the (*ex ante*) profit/loss

on day $t+1$. It is added into a separate bank account. The slate is then wiped clean and we start on day $t+2$ again with a zero-investment. The point of this zero-investment procedure is that we always start from a position of \$0.00, so today's investment does not depend on yesterday's position. Thus it is time homogeneous and comparability between days is maintained. This procedure also allows comparability between investment strategies which by construction necessitate different absolute overall investments in the risky assets.

Ex ante evaluations will be reported for all 30-day windows in the form of cumulated excess returns up to time $t + 1$, $36 \leq t + 1 \leq 1517$, i.e.⁵

$$R_{(36,t+1)}^{(ea)} = \sum_{s=36}^{t+1} R_s^{(ea)}. \quad (8)$$

The evaluations are made in terms of achieved standard deviations and Sharpe ratios, as well as in terms of returns.

We take as null hypothesis that excess returns from any strategy will average to zero. The portfolio selection strategies we implement are not risk adjusted, and may have quite different systematic risks. The *a priori* expectations here are perhaps not obvious, but the standard method requires shorting the entire portfolio and rebalancing when $\widehat{TPC} \leq 0$,

⁵ Because our ex-ante results allow a one-day lag between portfolio allocation and return calculation and the averaged back (augmented) portfolio requires a lead-up of 4 augmented portfolio observations,, these evaluations begin on May 8, 1996, giving a total of $T - 29 - 4 - 2 = 1482$ observations.

whereas the combined method can be applied continuously, so we might postulate the latter to be the less variable of the two.

As another measure of the frequency and magnitude of rebalancing necessary to a strategy, we assess *transaction costs*. For our purposes these are defined as follows. The overall portfolio can be considered as a combination of individual leveraged positions in each risky asset. At the end of day $t + 1$, the investment in leveraged asset i has value $w_t^{(i)}(\tilde{R}_{(t,t+1)}^{(i)} - r_{t+1})$, which is then rebalanced to a value of zero ($w_{t+1}^{(i)}$ in asset i and $-w_{t+1}^{(i)}$ in the risk-free asset) at the start of day $t + 2$. A transaction cost measure was obtained as a fraction of the sum of the absolute level of rebalancing required for each leveraged component of the overall portfolio:⁶

$$T_{t+1} = 0.005 \sum_{i=1}^N |w_t^{(i)}(R_{(t,t+1)}^{(i)} - r_{t+1})|.$$

5. Results

5.1. Ex Ante Returns

The time series in Figures 2a–2f show the daily accumulated amounts in the “bank account”, kept as a record of the rolling 30-day zero-investment procedure, for each of the

⁶ The multiplier 0.005 was arrived at as follows. Trading Spiders and iShares may be conducted via fixed or variable priced brokers by traders subject to a wide variety of tax regimes. Trading costs are, however, trivial. The multiplier 0.005, is arbitrary, but based on our estimate of what might be reasonable following discussions with market participants.

6 allocation schemes listed at the end of Section 3, together with the returns on the World Index (Figure 2g). The first panel (first six rows) of Table 1 list the relevant statistics for the bank account. (The next two sets of rows in Table 1 list transaction costs, and the bank account adjusted for transaction costs, respectively.)

[FIGURE 2 ABOUT HERE]

(i) The *unadjusted augmented Markowitz method* ends the period overall with a gain of +\$42.23 over the initial \$0 investment (Column 1 of Table 1). Recall that these gains are excess over the risk-free rate. Since the optimisation process produces an investment in the risky assets of $\pm\$1$ for the augmented and combined Markowitz methods, the gain can be thought of as a return per \$1. The variability is high (daily return standard deviation of 0.72318), a reflection of the high variability in portfolio weights illustrated in Figure 1b and 1c, though not as high perhaps as we might be led to expect from those figures. The overall mean/standard deviation ratio was 0.03940 (Column 1 of Table 1).

[TABLE 1 ABOUT HERE]

(ii) For the *adjusted portfolios*, we see progressively less variable and correspondingly lower returns with the *averaged back, weighted back and Lintner smoothed methods*, clearly exemplifying the return-risk tradeoff. The mean/standard deviation ratios get progressively lower at 0.03993, 0.02605, 0.00839 (Columns 2, 3, 4 of Table 1).

(iii) The *unsmoothed combined method* is considerably less variable than the augmented Markowitz method (daily return standard deviation of 0.22239), as conjectured in Section 4, but the corresponding cumulative return is much lower at \$4.87, leading to a

lower mean/standard deviation ratio: 0.01476 (Column 5 of Table 1).

(iv) Cumulated returns for the *naive method* (Figure 2f) are lower than for the Lintner smoothed method (Figure 2e); note the huge change of scale in going from Figures 2a–2d to Figures 2e–2f.

(v) An interesting comparison is with the *World Index* (Figure 2g). (The World Index is plotted on the same scale as the Lintner smoothed and naive results.) Over the whole period, the world produces a paltry return of \$0.063 (Column 7 of Table 1), together with a very poor mean/standard deviation ratio of 0.00491, only a little better than the naive ratio of -0.0034 (Column 6 of Table 1).

Regarding significance, the augmented Markowitz method returns an overall average of $+0.0285$, which, divided by its standard error of $0.7232/\sqrt{1482}$, gives a ratio of $+1.52$. According to our statistical discussion in Appendix 3, this can be taken approximately as a normal random variable and consequently is not significantly different from 0. Similarly, none of the other methods produces a significant return, overall. Nevertheless we are struck by the fact that all the Markowitz or scaled Markowitz methods in Figures 2a–2f spend most of the time in positive territory. While this is not statistically significant⁶ it

⁶ The area under a standard Brownian motion over the period $[0, 1]$, i.e., $\int_0^1 W_t dt$, is distributed as $N(0, 1/6)$, and by a functional central limit theorem for weakly dependent variables (Herrndorf, 1984) this will be approximately true also in large samples, such as we have, for the area under a normalised sum of m -dependent random variables. For the augmented Markowitz method, which is the most striking in this respect, this statistic has

is indicative that the Markowitz methods are producing results in the right direction.

5.2. Transaction Costs

The second panel in Table 1 shows the transaction costs cumulated over the period of the study, and the third panel has returns net of transaction costs for the 6 allocation versions under study. As predicted, the “combined” method incurs less cost (\$2.87 as compared with \$3.92) but the difference is small by comparison with the difference in the overall returns on those two strategies. The transaction costs for the averaged and weighted portfolios are, surprisingly, rather high, while that for the Lintner scaled is low, as is the case for the naive. But the returns on the last two are so small that the transaction costs are enough to drive the overall (excess) returns (slightly) negative.

6. Discussion

6.1. The TPC

Our results show that TPC is a relevant object to study in the Markowitz theory. This issue seems to have been given scant attention in the literature or in practise, previously. One school of thought is: $TPC \leq 0$ doesn't happen! (e.g., Constantinides and Malliaris 1995). But empirical evidence is definitive on this point: $TPC \leq 0$ occurs frequently in data. For example, in our (large) set, which mirrors international markets, it occurs persistently over significant periods of time, and there is no evidence that any kind of stationary or equilibrium regime is being approached (Figure 1a).

the value +1.62, which is not significant as a $N(0, 1)$ value.

The second textbook response is: $TPC \leq 0$ doesn't happen! (but if it does, then we should short the usual Markowitz portfolio and invest the proceeds in the risk-free asset). A risk-free asset is not always available, but assuming that one is, the *a priori* objection to this response is again suggested by Figure 1a: clearly there will be periods when frequent and massive portfolio re-balancing (shorting the entire portfolio then buying it back again, in rapid succession) would be needed to implement such a strategy. It would seem, *a priori* that the transaction costs involved in this would surely be prohibitive. It was surprising to us, and should be enlightening to practitioners, that this turned out not to be the case at all, in our investigation. The reason, as can be seen from Figure 1a, is that conditions where $TPC < 0$ occurs tend to be approached fairly smoothly, at least on a time scale measured in days, so that our quasi-continuous-time (daily) rebalancing moves the portfolio asset allocations relatively smoothly in the required direction. The other methods we tested are even less prone to large changes of asset allocations, but the overall beneficial effect of the Markowitz methodology was enough to outweigh any transaction cost penalty it incurred – at least in our data set. In a situation where rebalancing is done less often, this may not be so.

6.2. The Zero-Investment Procedure

An important and useful feature of our zero-investment assessment procedure is that it is time-homogeneous. Thus one can think of re-starting the procedure at any point in time and obtaining the same results with just a shift of origin to the value of the bank account at that time. For example, if we start the Lintner smoothed strategy at November

24, 1998, at which the bank account for that strategy is at its relatively high value of \$0.1343, the bank account for that strategy will contain -\$0.0859 at March 28, 2002, being $-\$0.1343 + \0.0484 , where \$0.0484 is the value of the bank account at March 28, 2002.

Correspondingly, if one had started the Lintner smoothed strategy at March 24, 2000, when the World Index is at its highest value, \$0.4616, the bank account for the Lintner smoothed strategy would contain \$0.0260 at March 28, 2002, being $\$0.0484 - \0.0224 .

7. Conclusion

Given that passive management is often advocated as a response to the challenge of managing equity in informationally efficient markets, the findings in this paper are striking. Our analysis shows that the quasi-continuous-time implementation of the augmented unadjusted Markowitz procedure (where the standard Markowitz procedure was augmented by shorting the portfolio and placing funds in the risk free asset when the risk premium was negative) generated the largest dollar gain and the second highest empirical Sharpe ratio. While the transaction costs generated using this strategy were higher than those of the other strategies examined, those extra costs were not of a magnitude that would make the unadjusted Markowitz approach less attractive than its competitors. Clearly, in our sample, the historically derived inputs were by no means stale. The augmented unadjusted Markowitz procedure seems to have “captured” sufficient information to function well. Other methods, albeit *ad hoc*, which we introduced with a view to reducing noise in the *ex post* derived inputs seemed to subtract, rather than add, value to our allocations.

Optimising appears to be the optimal strategy.

Our findings appear robust to “bear” conditions (where $TPC < 0$) and, as such, should be heartening to practitioners. The difference between the augmented unadjusted Markowitz strategy and the combined unadjusted strategy was surprising to us given the frequency of negative TPC episodes and the use of a technique explicitly accounting for these observations (Maller and Turkington, 2002). While this analysis is not a thoroughgoing examination of the procedure advocated by Maller and Turkington, the preliminary empirical analysis reported in this paper is not encouraging as to its value, in general.

A clinical study cannot of course guarantee out-of-sample performance such as that found in the present paper. But the bottom line conclusion of our analysis is nonetheless clear. Using a realistic data set we have shown that a quasi-continuous-time implementation of the Markowitz procedure, shorting stocks and holding the risk free asset when the risk premium is negative, resulted in superior (economically significant, though of course not statistically significant) raw *ex ante* returns. Investors should consider the implications of such a strategy. In both bull and bear conditions, this strategy, or some modification of it, represents an attractive alternative to passive investment in informationally efficient markets

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provided some background for the present paper. We are very grateful for their assistance. We also thank Kent Zumwalt for helpful discussions. In order to test a procedure that is readily implementable in practise, the calculations for this paper were all done on EXCEL. Accuracy was checked in various ways. In particular, we are grateful to Berwin Turlach for cross-checking some of our matrix calculations against Splus results.

Appendix 1. Static Markowitz Optimisation: Augmented & Combined Methods

We have N risky assets, whose expected daily (discrete) returns are collected into the N -vector $\tilde{\mu}$. (The tilde is used to denote raw returns; μ will be used for excess returns.) We also have a risk-free asset returning r , say. Portfolio allocation is specified by an N -vector \mathbf{x} , whose components add to 1 (total allocation constraint). The expected return on the portfolio is then $\mathbf{x}'\tilde{\mu}$, with variance $\mathbf{x}'\Sigma\mathbf{x}$, where Σ is the $N \times N$ covariance matrix of the risky asset returns (the prime denotes a vector or matrix transpose). Following the textbook procedure (e.g., Markowitz 1952, 1991, Elton and Gruber 1997, Grinold and Kahn 2000), an efficient frontier of optimal (mean-variance efficient) portfolios can be found in risk-return space.

A single point can be selected on the frontier by specifying an investor's risk preferences in some way. Another way of selecting a unique portfolio with optimal risk-return properties, independent of risk preferences, is via the *tangent point* from the risk-free asset coordinates to the efficient frontier. *When it exists* on the upper part of the possibility set of portfolios, the tangent point gives the portfolio with *maximum Sharpe ratio* – the

maximum expected excess return for specified risk possible from the data. Equivalently, this portfolio maximises the function

$$f(\mathbf{x}) = \frac{\mathbf{x}'(\tilde{\mu} - r\mathbf{i})}{\sqrt{\mathbf{x}'\Sigma\mathbf{x}}} = \frac{\mathbf{x}'\mu}{\sqrt{\mathbf{x}'\Sigma\mathbf{x}}}$$

for variations in \mathbf{x} under the total allocation constraint $\mathbf{i}'\mathbf{x} = 1$. Here \mathbf{i} is an N -vector each of whose elements is 1, and $\mu = \tilde{\mu} - r\mathbf{i}$ contains the expected *excess returns* of the assets. We allow short sales of assets, so the components of \mathbf{x} may take either sign, and, as advocated by Sharpe (1994), we maximise $f(\mathbf{x})$ with regard to sign.

However, in periods of low or negative returns, the standard Markowitz procedure specifies a portfolio on the lower (sub-optimal) portion of the efficient frontier. In this case the portfolio with maximum Sharpe ratio cannot be obtained as a tangent point to the efficient frontier – instead, the maximum has to be found from a limiting procedure (Maller and Turkington 2002). We can distinguish the two cases with the “tangent point criterion” defined as follows:

$$TPC := \frac{\mathbf{i}'\Sigma^{-1}\mu}{\sqrt{(\mathbf{i}'\Sigma^{-1}\mathbf{i})(\mu'\Sigma^{-1}\mu)}}. \quad (1)$$

The *TPC* gives a very specific interpretation to what we mean by a positive or negative “risk premium”. Suppose the *TPC* is positive. Then a straightforward analysis shows that the tangent point allocation is given by

$$\mathbf{x}_{TP} = \frac{\Sigma^{-1}\mu}{\mathbf{i}'\Sigma^{-1}\mu} \quad (2)$$

with a corresponding (positive) Sharpe ratio of

$$+\sqrt{\mu'\Sigma^{-1}\mu}. \quad (3)$$

If, however, $TPC < 0$, then the allocation (2) gives the (sub-optimal) *minimum* Sharpe ratio, having value the negative of that in (3). In this case the maximum Sharpe ratio achievable from the risky assets is shown by Maller and Turkington (2002) to have value

$$+\sqrt{\mu'\Sigma^{-1}\mu - (\mathbf{i}'\Sigma^{-1}\mu)^2/\mathbf{i}'\Sigma^{-1}\mathbf{i}}, \quad (4)$$

which is non-negative, but smaller than that in (3).

But whatever the sign of TPC , the Sharpe ratio in (3) can be achieved with the allocation in (2) (or its negative) if we also allow risk-free lending and borrowing. In effect, this adds the risk-free asset to the portfolio. The strategy is as follows. Invest a units of resource in the risky portfolio, having risky assets allocated according to x , and $1 - a$ units in the risk-free asset. There is still an overall allocation of 1 unit, but we allow $a < 0$ or $a > 1$. For such a portfolio, the expected return is

$$a\mathbf{x}'\tilde{\mu} + (1 - a)r = a\mathbf{x}'\mu + r,$$

so the expected excess return is $a\mathbf{x}'\mu$, and the standard deviation is

$$\sqrt{(a\mathbf{x})'\Sigma^{-1}(a\mathbf{x})} = |a|\sqrt{\mathbf{x}'\Sigma^{-1}\mathbf{x}}.$$

Choosing $\mathbf{x} = \mathbf{x}_{TP}$ (whether or not $TPC \geq 0$) gives a Sharpe ratio of

$$\text{sgn}(a)\text{sgn}(TPC)\sqrt{\mu'\Sigma^{-1}\mu}. \quad (5)$$

Then by choosing the sign of a appropriately, we can achieve the Sharpe ratio in (3), the maximum possible.

Thus, if $TPC > 0$ (the standard case) we can take any $a > 0$ in the new portfolio and achieve the Sharpe ratio in (3) by taking $\mathbf{x} = \mathbf{x}_{TP}$ as in (2). In particular, if $a = 1$ we have all the investment in the risky assets, allocated according to (2), and none in the risk-free asset. If on the other hand $TPC < 0$ we can achieve (3) by taking \mathbf{x}_{TP} as in (2) (this will give the sub-optimal minimum Sharpe ratio allocation) and $a < 0$ in the new portfolio. For example, we could take $a = -1$, corresponding to 2 units invested in the risk-free asset and a total of -1 unit in the risky assets, i.e., a short position is taken in the minimum Sharpe ratio allocation. The choice of a here is not unique; any other negative value of a will achieve the same (maximum) Sharpe ratio, but with some other risky versus riskless asset assignment.

We might hypothesise *a priori* that, since the augmented standard method requires shorting the entire portfolio and rebalancing when $\widehat{TPC} \leq 0$, whereas the combined method can be applied continuously, the latter might be the less variable of the two, and require less rebalancing and hence less transaction costs; thus, despite being suboptimal, it may still be the more economic.

The above analysis shows that it is the *sign* of TPC which is important; the denominator is included in (1) so as to standardise the measure between -1 and 1 , so that it has characteristics something like those of a correlation coefficient. Strictly speaking, TPC does not measure just the positivity or otherwise of (excess) *returns*, although it is closely related to this; it also takes into account in a certain way *the structure of the covariance matrix* of the returns. It is shown in Maller and Turkington (2002) that $TPC \leq 0$ can

occur even with all components of μ positive, as a result of a particular conformation of Σ . This situation could well occur in a real data set.

Appendix 2. Portfolio Definitions, Transaction Costs Formula

The portfolios prescribed by the three methods are obtained as follows:

Augmented Markowitz Method:

$$\mathbf{x}_t(A) = \begin{cases} \frac{\Sigma^{-1}\mu}{\mathbf{i}'\Sigma^{-1}\mu}, & \text{if } TPC > 0 \\ -\frac{\Sigma^{-1}\mu}{\mathbf{i}'\Sigma^{-1}\mu} & \text{if } TPC \leq 0 \end{cases} \quad (A.1)$$

(where \mathbf{i} is an N -vector of 1s).

Combined Method:

$$\mathbf{x}_t(C) = \begin{cases} \frac{\Sigma^{-1}\mu}{\mathbf{i}'\Sigma^{-1}\mu}, & \text{if } TPC > 0 \\ \left(\lambda u_{\max}, 1 - \lambda \bar{\mathbf{i}}' u_{\max}\right), & \text{if } TPC \leq 0, \end{cases} \quad (A.2)$$

where u_{\max} is as specified in Maller and Turkington (2002), $\bar{\mathbf{i}}$ is an $(N-1)$ -vector of 1s and $\lambda > 0$ is a factor by which the Sharpe Ratio (SR^C , say) of $\mathbf{x}_t(C)$ approaches its maximum as λ increases. We choose λ so large that:

$$\frac{SR^C(\lambda + 0.00001) - SR^C(\lambda)}{0.00001} = 0.01.$$

Naive Method:

$$\mathbf{x}_t^{(i)}(N) = \left(\frac{1}{N}\right) \mathbf{i}. \quad (A.3)$$

Appendix 3. Statistical Aspects

Apart from the usual considerations, there are some particular statistical problems to be taken into account when assessing returns variabilities and comparing *ex ante* returns (as is our focus) between methodologies. Neither the *ex post* nor the *ex ante* returns as calculated from the estimated portfolio allocations are independent in time (over the index t), because of the overlap in the moving window of width 30 days. For this reason, some of the usual statistical procedures, even involving such standard methods as histograms, QQ plots, etc., cannot be directly applied. Some tests can be justified, however, at least in large samples such as we have here. For the *ex post* returns, a correct adjustment for the dependence would be complicated to apply, but we are not interested in statistical comparisons of these. In the case of the *ex ante* returns, where we do wish to make comparisons, the situation is much simpler. The portfolio allocations are calculated from past observations, while the *ex ante* returns look into the future. Thus they form a stationary martingale difference series (under the null hypothesis that the raw returns are observations on a mean zero stationary sequence), and the martingale central limit theorem can be invoked to justify comparisons of *ex ante* mean (excess) returns either with zero (an efficient market test), or with each other (for portfolio performance comparisons), using the usual t -statistics. These will be distributed close to standard normal. In fact, more is true: the *ex ante* excess returns are observations on “ m -dependent” random variables, i.e., \mathbf{R}_i and \mathbf{R}_j are independent if $|i - j| > m$, where here $m = 32$. We can use the limit theory worked out for these in Herrndorf (1984) to justify the significance tests.

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LEGENDS TO FIGURES

Figure 1a: The observed TPC over the operating period May 7, 1996–March 28, 2002 (1487 daily observations), calculated from $\hat{\mu}_t$ and $\hat{\Sigma}_t$ using Eq. (1) of Appendix 1. Negative values of TPC occur about 40% of the time.

Figures 1b and 1c: The values of L_t over the operating period May 7, 1996 – March 28, 2002 for the augmented and combined methods, respectively.

Figure 1d: The determinant D_t of $\hat{\Sigma}_t$ over the operating period May 7, 1996 – March 28, 2002.

Figures 2a–2f: The daily accumulated returns from the rolling 30-day zero-investment procedure, for each of the 6 allocation schemes listed at the end of Section 3, together with the returns on the World Index (Figure 2g).

Table 1: Basic statistics for 6 portfolio schemes, and the World Index.

Fig 1a: Time Series of TPC Observations

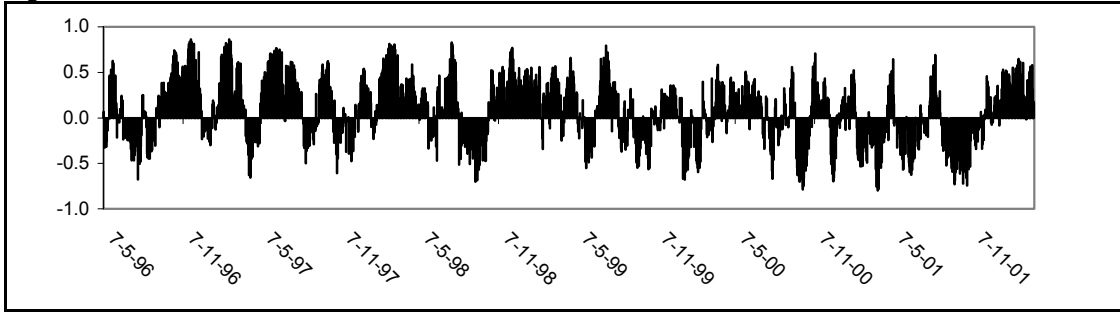


Fig 1b: Time Series of Lintner Scores, augmented Markowitz, unadjusted portfolios

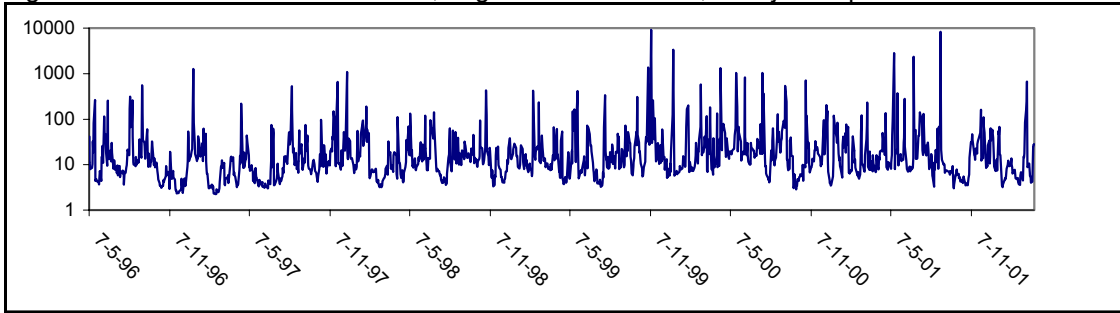


Fig 1c: Time Series of Lintner Scores, combined method, unadjusted portfolios

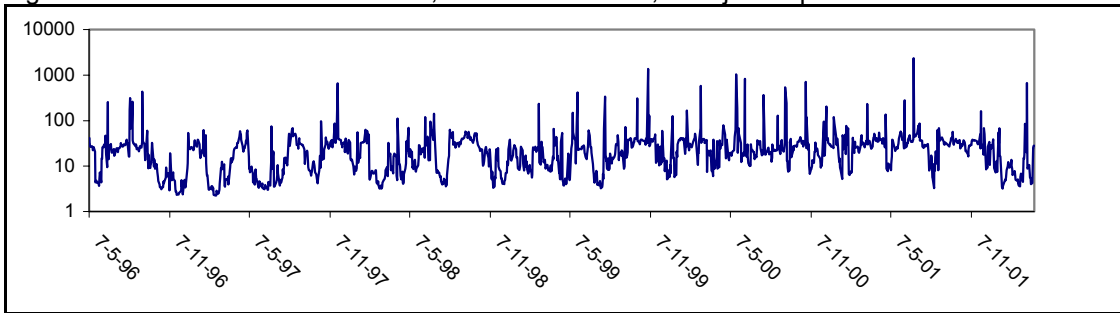


Fig 1d: Time Series of Covariance Matrix Determinants

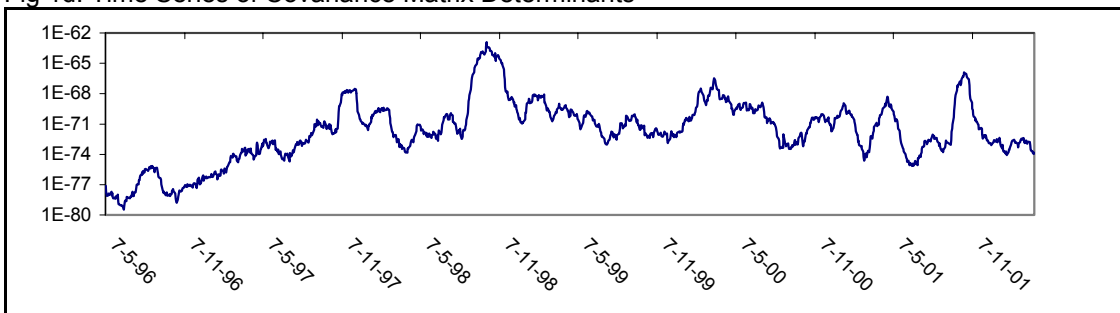


Fig 2a: Ex ante daily (RHS) and cumulative (LHS) returns, augmented Markowitz, unadjusted portfolios

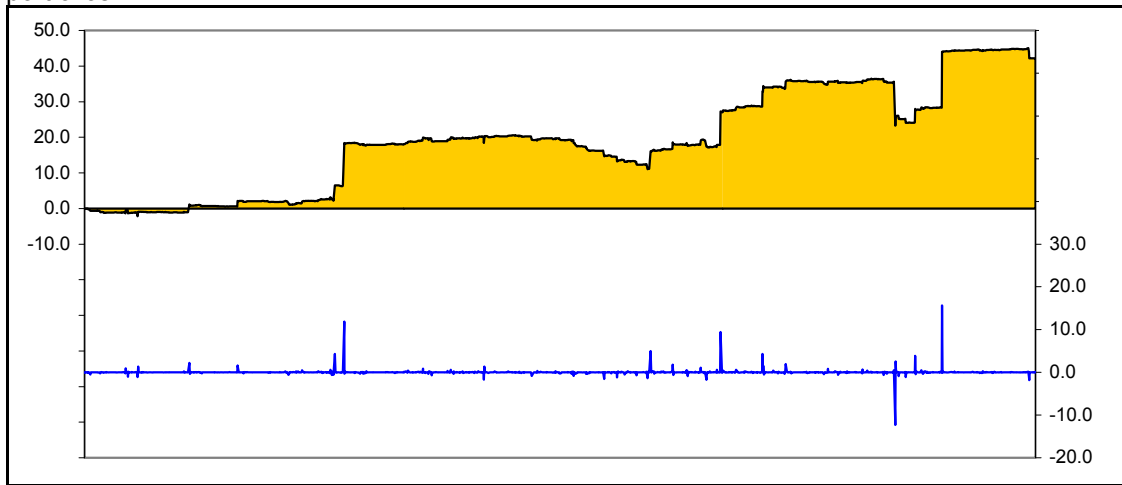


Fig 2b: Ex ante daily (RHS) and cumulative (LHS) returns, augmented Markowitz, averaged back portfolios

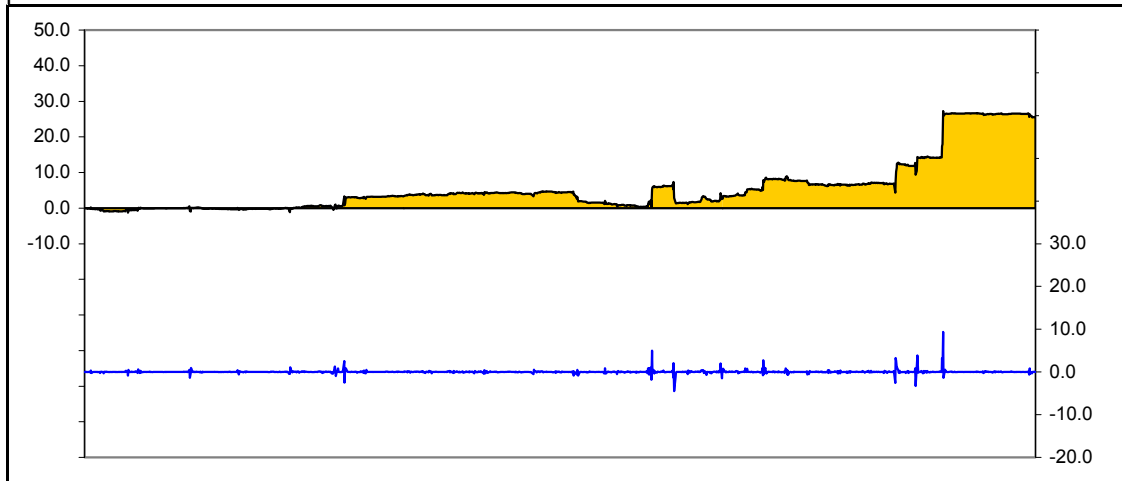


Fig 2c: Ex ante daily (RHS) and cumulative (LHS) returns, augmented Markowitz, weighted back portfolios

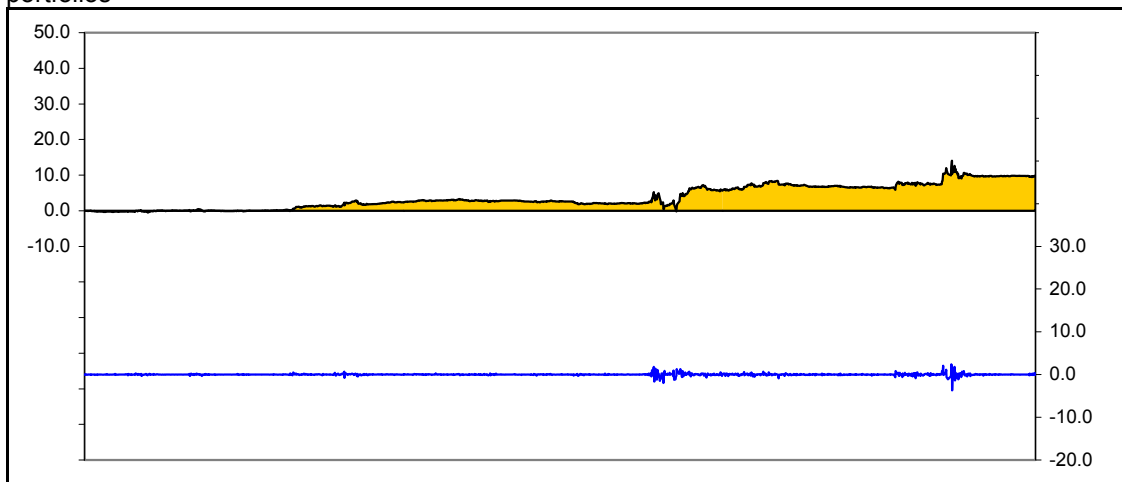


Fig 2d: Ex ante daily (RHS) and cumulative (LHS) returns, combined method, unadjusted portfolios

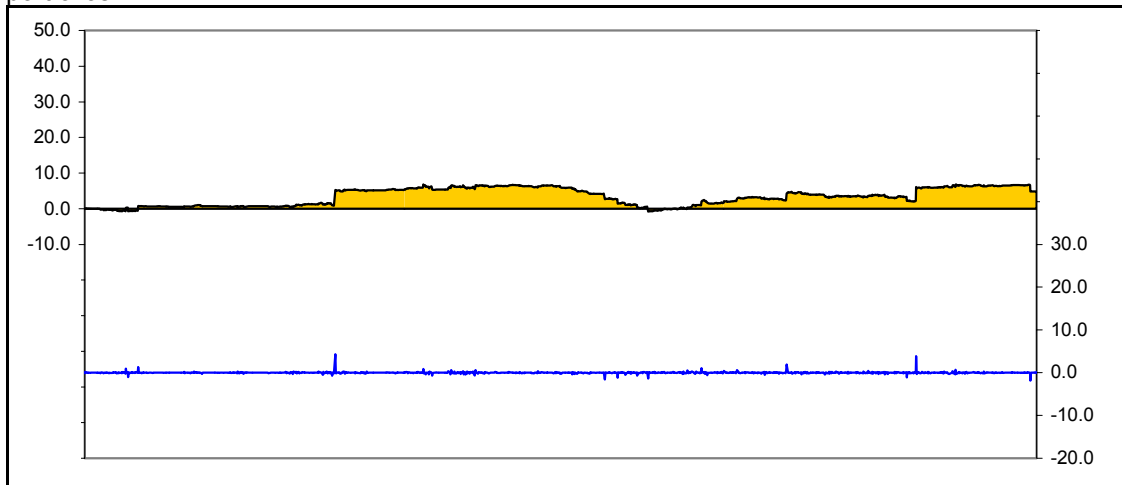


Fig 2e: Ex ante daily (RHS) and cumulative (LHS) returns, augmented Markowitz, Lintner scaled portfolios

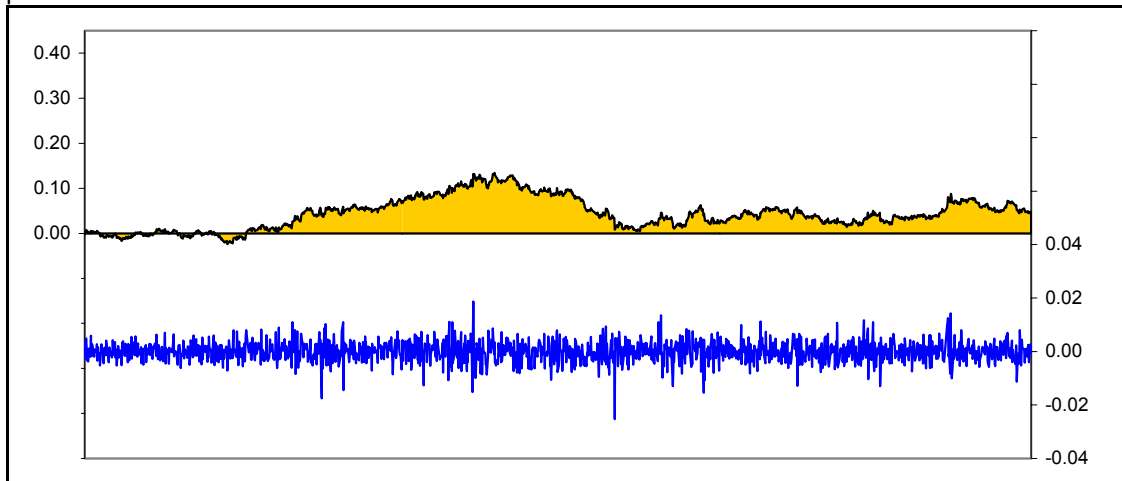


Fig 2f: Ex ante daily (RHS) and cumulative (LHS) returns, naïve portfolios

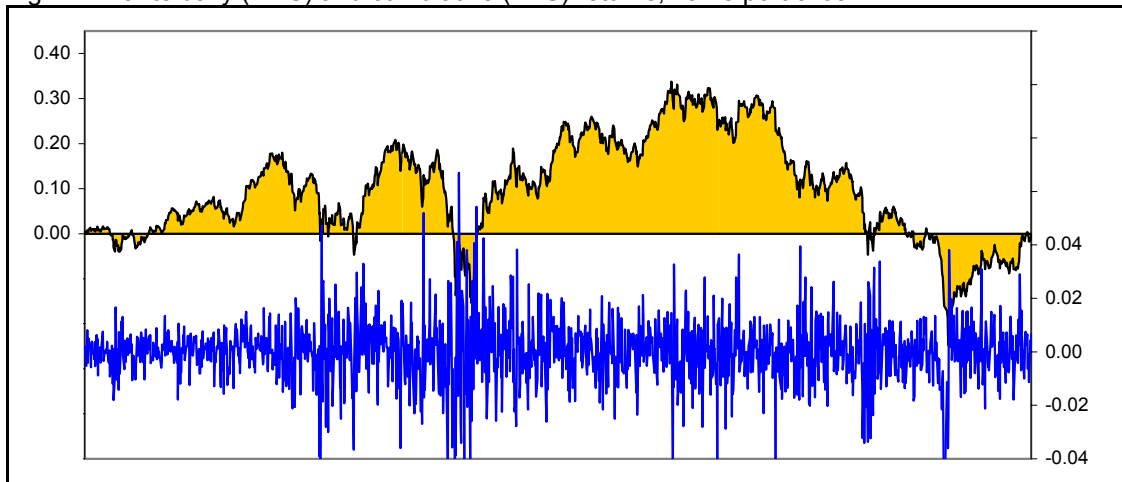


Fig 2g: Ex ante daily (RHS) and cumulative (LHS) returns, world index

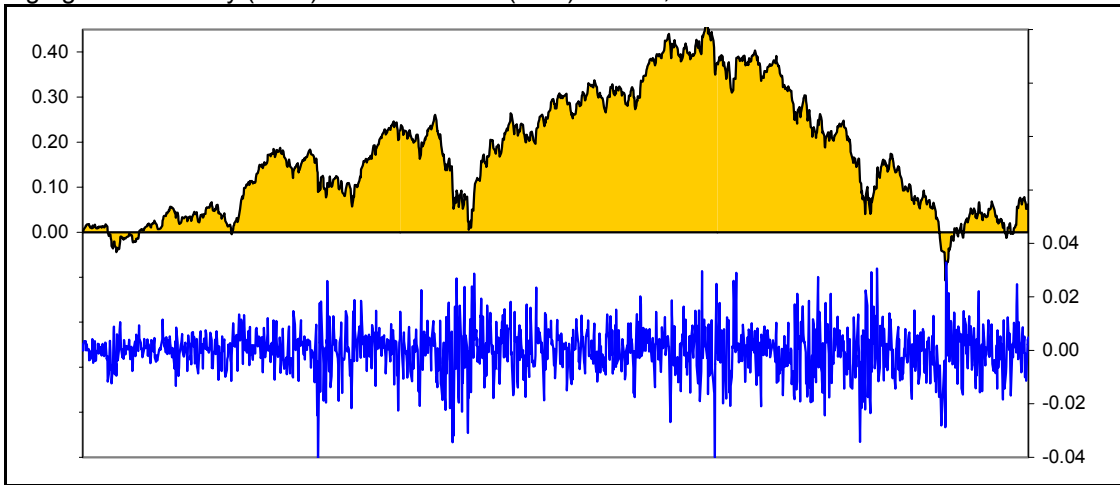


Table 1 Returns Summary Statistics, Ex Ante Evaluation of 7 Portfolio Schemes

t: 7/5/96 to 27/3/02 (1482 observations)		Augmented Markowitz				Combined	Naive	World Index
		Unadjusted	Averaged Back	Weighted Back	Lintner Scaled	Unadjusted		
μ_{t+1}	min	-12.30337	-4.51982	-3.68448	-0.02526	-1.83870	-0.06323	-0.04342
	max	15.68001	9.32643	2.37232	0.01872	4.33812	0.06694	0.03286
	mean	0.02849	0.01726	0.00660	0.00003	0.00328	0.00000	0.00004
	st dev	0.72318	0.43213	0.25336	0.00390	0.22239	0.01171	0.00863
	mean/st dev	0.03940	0.03993	0.02605	0.00839	0.01476	-0.00034	0.00491
	cumul	42.22857	25.57428	9.77958	0.04843	4.86497	-0.00591	0.06277
T_{t+1}	min	0.00006	0.00007	0.00010	0.00001	0.00006	0.00002	
	max	0.40879	0.16564	0.09356	0.00031	0.10269	0.00035	
	mean	0.00264	0.00302	0.00240	0.00006	0.00194	0.00007	
	st dev	0.01452	0.01051	0.00479	0.00003	0.00441	0.00004	
	mean/st dev	0.18179	0.28730	0.50065	1.79859	0.43925	1.86448	
	cumul	3.91099	4.47384	3.55372	0.09308	2.86942	0.09725	
$\mu_{t+1} - T_{t+1}$	min	-12.42987	-4.59843	-3.71811	-0.02546	-1.87250	-0.06354	
	max	15.43937	9.24482	2.34714	0.01857	4.30345	0.06659	
	mean	0.02586	0.01424	0.00420	-0.00003	0.00135	-0.00007	
	st dev	0.71648	0.43046	0.25338	0.00390	0.22094	0.01171	
	mean/st dev	0.03609	0.03308	0.01658	-0.00773	0.00609	-0.00595	
	cumul	38.31758	21.10044	6.22585	-0.04465	1.99555	-0.10316	